# Explaining Control Policies through Predicate Decision Diagrams

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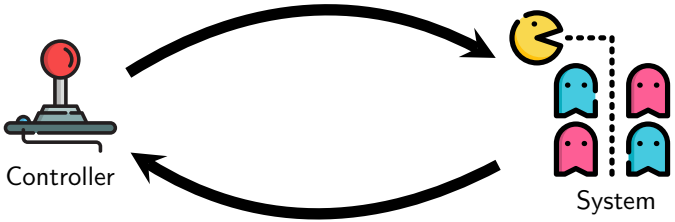
Hybrid Systems: Computation and Control
Irvine, California, May 2025

# Controller



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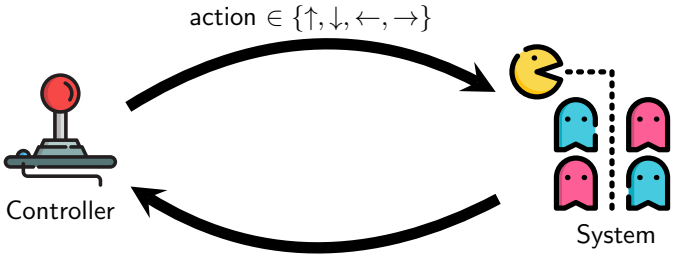
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Controller
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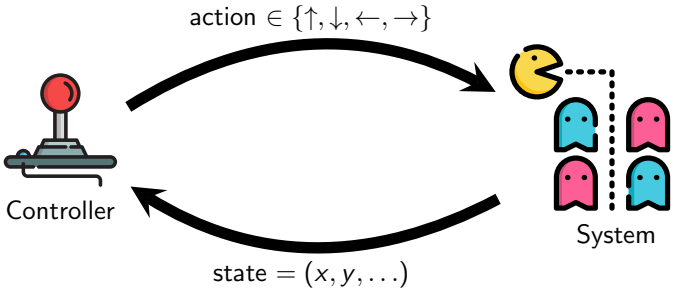
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Controller
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state $= (x, y, \ldots)$

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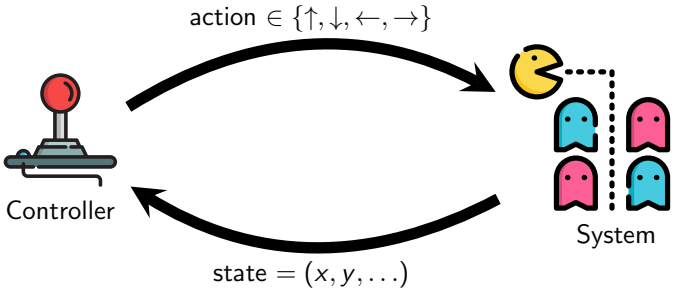


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```
⋮

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57049 0 1 3 4 6 7
57050 0 1 3 4 6 7
57054 2 5
57055 1 2 4 5
57056 1 2 4 5
57057 0 1 2 3 4 5
57058 0 1 2 3 4 5
57059 0 1 2 3 4 5
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- Lacks explainability

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  - **Linear**: $x + y < 7$
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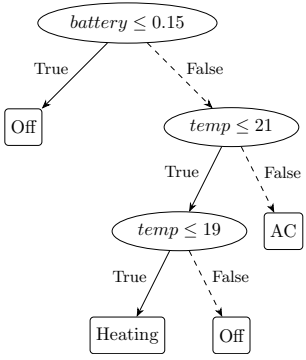
- Decision Trees: Explainability through predicates

# Decistion Trees (DT)

- Binary tree
- Internal nodes = predicates
- Branches = True/False outcome of that predicate
- Leaf nodes = actions



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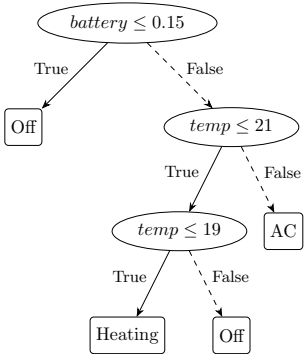


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✗ Often redundant due to repeated subtrees

# Binary Decision Diagrams (BDD)

- Directed acyclic graph
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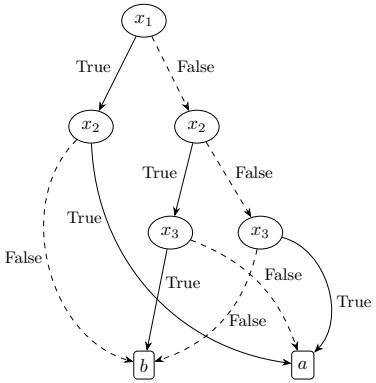
- Reduced BDD: No redundant nodes
- Ordered BDD: Fixed variable order



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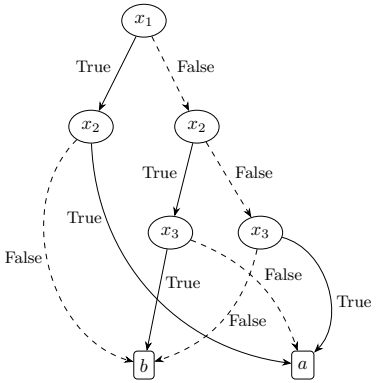

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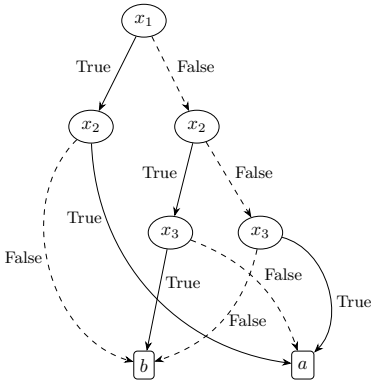

# Explainable Control Policy

**Decision Trees**

- ✓ Easy to visualize and interpret
- ✗ Not suitable for symbolic operations
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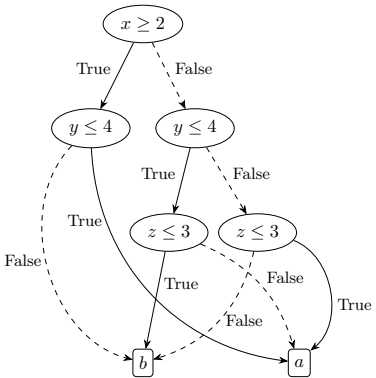
🎯 **Our goal** : Find a representation that is:

- ✓ Uses predicates over variables like DTs $\rightsquigarrow$ explainable
- ✓ Supports BDD-based algorithms
- ✓ Avoids bit-blasting; stays at semantic predicate level

# Predicate Decision Diagrams (PDD)

- Extends BDDs with predicates



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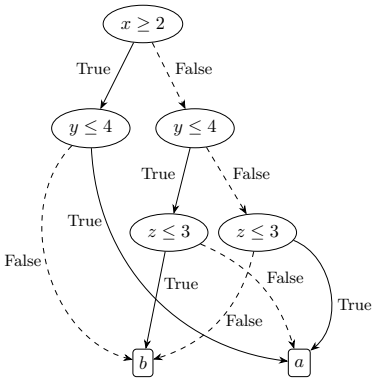

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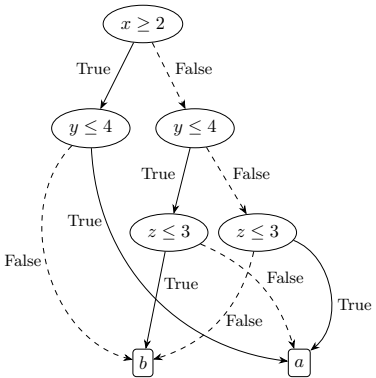

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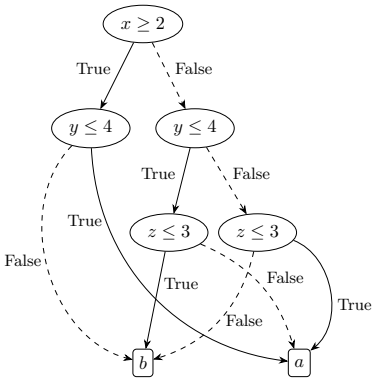
- ✓ Merges isomorphic subgraphs, reducing redundancy
- ✓ Compact representation (similar to BDDs)
- ✓ Avoids bit-blasting: operate directly on predicates over state variables



# How to Synthesize a PDD?

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Controller Policy Table

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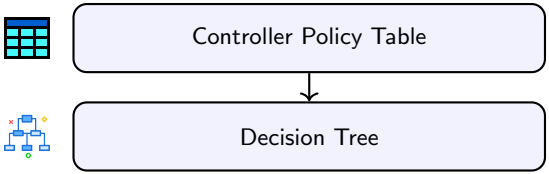

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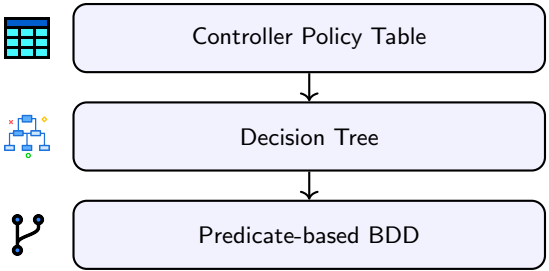

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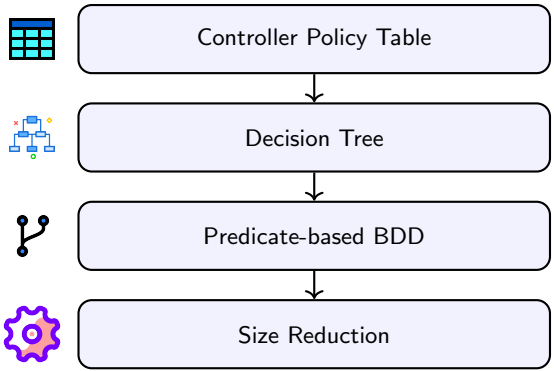

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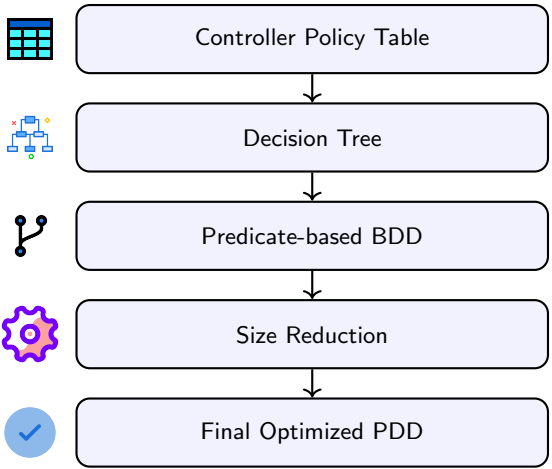

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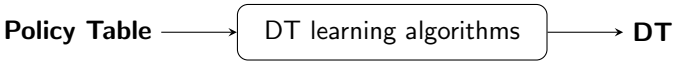

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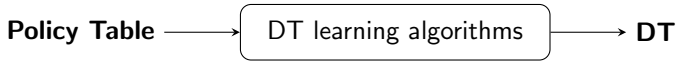


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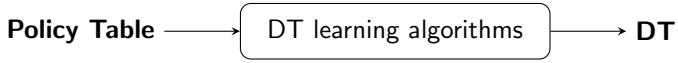


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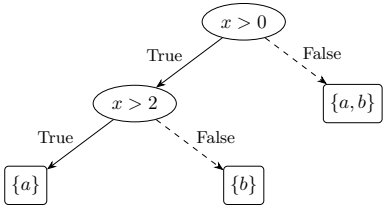

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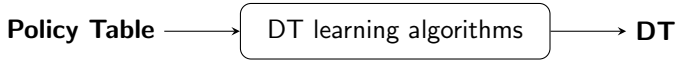


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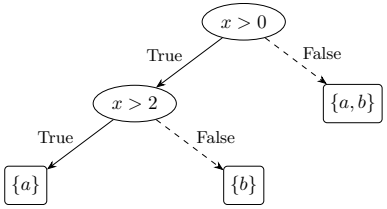


- Learn DT from the policy
- No early stopping : completely capture the policy

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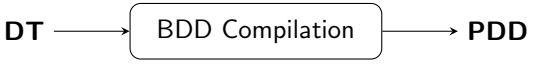


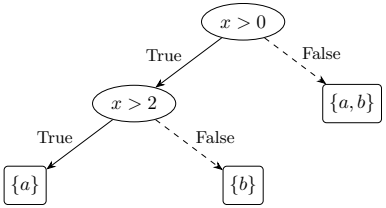

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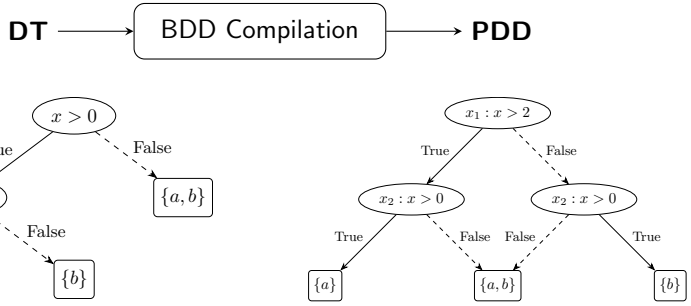


- Each predicate corresponds to a Boolean variable
- Reduce and merge to get canonical ROBDD

# Phase 3 : Size Reduction by Consistency Checking

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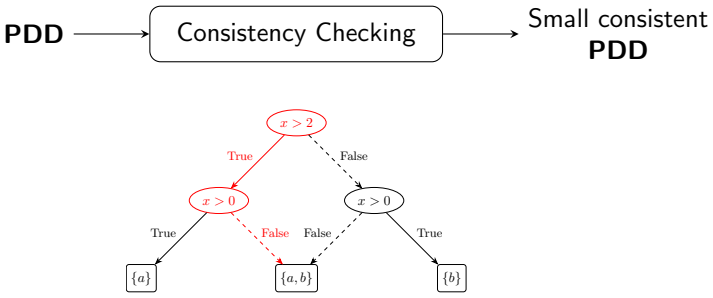


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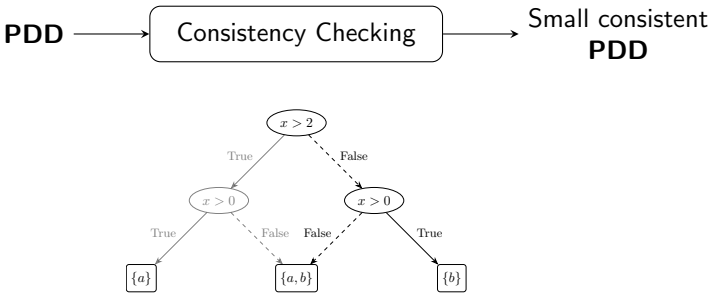


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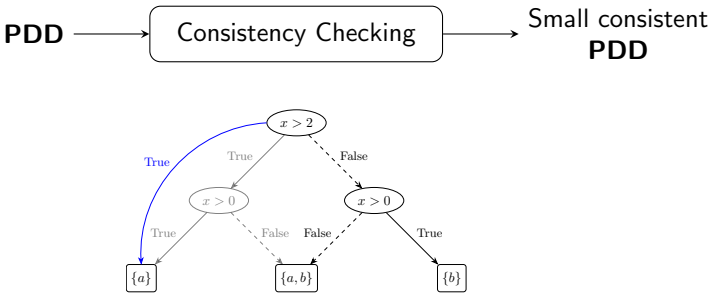


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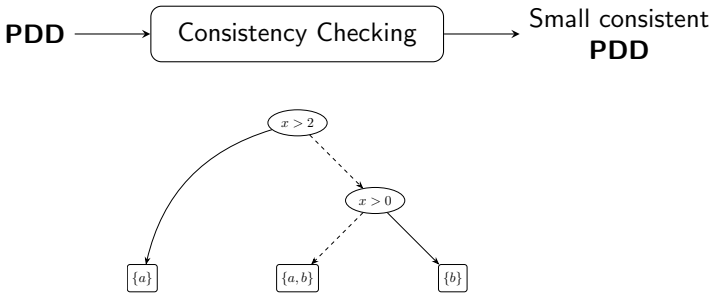


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  - Goal: Find a good predicate order to reduce size.
  - Uses *sifting* algorithm[1].
- **Care-set reduction**
  - Unreachable state $\rightsquigarrow$ action does not matter
  - Compresses the diagram by only preserving outputs for relevant states.
  - Based on *restrict* operator[2].

[1] Rudell, R. "Dynamic variable ordering for ordered binary decision diagrams", ICCAD'93.
[2] Coudert, & Madre. "A unified framework for the formal verification of sequential circuits", ICCAD'90.

# Experiments

**Benchmarks**:

- Cyber-physical systems :
  - Benchmarks from SCOTS [3] and UppAal [4]
- Markov Decision Processes:
  - Quantitative Verification Benchmark Set (`https://qcomp.org`)
  - Policy extracted using Storm[5]

Implemented in python: dtControl + BuDDy

[3] Rungger, M, and Zamani M. "SCOTS: A tool for the synthesis of symbolic controllers." HSCC'16.
[4] David, A, et al. "Uppaal stratego." TACAS'15.
[5] Dehnert, C, et al.. "A storm is coming: A modern probabilistic model checker." CAV'17.

# Results: Size Comparison

Table: Selected results: PDD vs. DT and bit-blasted BDD sizes

| Controller | States | BDD | DT | PDD | Comments |
|---|---|---|---|---|---|
| `10rooms` | 26,244 | 1102 | 8648 | **344** | PDD much smaller |
| `helicopter` | 280,539 | 3348 | 3169 | **3158** | PDD has comparable size to DTs |
| `cartpole` | 271 | 197 | **126** | **126** | DT = PDD |
| `blocksworld.5` | 1,124 | 4043 | **617** | 796 | PDD more compact than the BDD |
| `pnueli-zuck.5` | 303,427 | **59,217** | 85,685 | 72,192 | PDD more compact than the DT |

- PDDs are on average 77% smaller than the bit-blasted BDDs
- PDDs are on average 16% smaller than the DTs

# Ablation Study

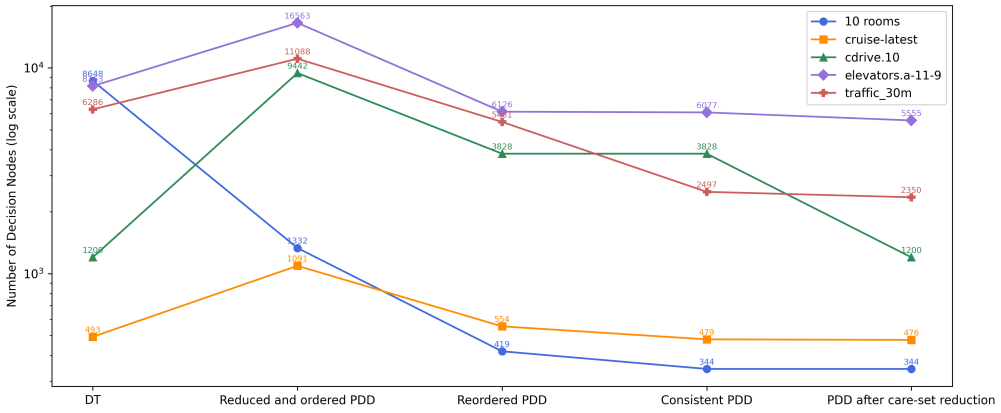


Figure: Effect of different steps in the pipeline in selected benchmarks

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